Fifty years ago, the concept of solitons in optical fibres was proposed numerically by F. Tappert and A. Hasegawa from Bell Labs. Seven years later in 1980, experiments confirmed that dispersion and the Kerr nonlinearity could combine to yield a coherent ultrashort pulse able to propagate without distortion. The implications of this discovery go well beyond the sector of telecommunications as initially suggested, and solitons now play a major role in modern ultrafast nonlinear photonics.
PIONEERING EXPERIMENT

OPTICAL SOLITONS

In the presence of nonlinearity, and the existence of the optical temporal soliton was predicted numerically in 1973 by Akira Hasegawa and Frank Tappert.

BALANCING DISPERSION AND NON-LINEARITY

The propagation of an optical wave in a single-mode optical fibre is governed by the 1D nonlinear Schrödinger equation or NLSE (see box) that takes into account the simultaneous interaction between dispersion and third-order (Kerr) nonlinearity [2]. When considered independently, dispersion and non-linearity are both detrimental to pulse propagation: dispersion will typically broaden the pulse temporally while nonlinearity will generate new frequency components and broaden the corresponding telecommunications potential, there was also interest in understanding the fundamental physics of how a short pulse of light would evolve if injected in a fibre at high power. This led to the emergence of a new theory of ultrashort pulse propagation

Figure 1. (a) Longitudinal evolution of the temporal (panels 1) and spectral (panels 2) properties of a fundamental soliton in the regime of anomalous dispersion. (b) Same as (a) but for a higher-order soliton (second order soliton). (c) Longitudinal evolution of the temporal profile of a black soliton in a fiber with normal dispersion. The numerical simulations are inspired by the one involved in [3], losses being not included.

Figure 2. Pioneer experimental results (black lines) from Fig 2 of Ref [4]. Input pulse characteristics (spectrum and autocorrelation) coupled into the fibre are depicted in the upper left box. Other graphs show the resulting pulse characteristics (spectrum, autocorrelation, temporal power profile) at fibre output for distinct input powers. Four distinct propagation regimes (columns a-d) were observed. Additional numerical simulations based on the NLSE are here provided for comparison (red dotted lines) using experimental parameters from Ref. [4]. Theoretical soliton solution (blue solid line) is shown for regime (b).
spectrum. In both cases, instantaneous frequencies are redistributed within the pulse (frequency chirp) and when dispersion and non-linearity act together, the temporal and spectral distortions can become highly complex. However, there is a special case: if the injected pulse has the temporal shape of a hyperbolic secant profile, the effect of non-linearity can exactly compensate the effects of dispersion, provided the pulse propagates in the anomalous dispersion regime so that the signs of the dispersive and nonlinear chirps are different. This yields a stationary wave able to propagate without distortion (Fig. 1a).

This is what Akira Hasegawa and Frederick Tappert (both with backgrounds in plasma physics) showed in 1973 [3]. Working at Bell Labs in the United States, they used theory and numerical simulation to consider the propagation of picosecond pulses with parameters typical of communication applications. They identified the bright ‘fundamental’ soliton as an ideal potential bit of information which would be robust against noise, collisions and other perturbations. However, they also recognized that the choice of power was crucial to achieve balance between dispersion and nonlinearity. Indeed, for higher powers, longitudinal periodic oscillations develop with alternating stages of temporal compression and splitting and broadening (Fig. 1b), consistent with theoretical predictions made by V. E. Zakharov and A. B. Shabat the year before in 1972. Significantly, Tappert’s numerical work was in itself pioneering, because solving the NLSE required that he developed a new and efficient method. This is now known as the split-step Fourier algorithm that remains the numerical tool of reference for the community 50 years later! In an accompanying publication, Hasegawa and Tappert showed that even propagation in the normal dispersion regime could yield stationary propagating structures, but this time they were “dark” solitons, a hole of light appearing in a continuous background (Fig. 1c).

**EXPERIMENTS**

Although the theory and numerics were clear in 1973, it took until 1980 before fibre solitons were seen in experiment, with the work also performed at Bell Labs by Linn F. Mollenauer, Roger H. Stolen and James P. Gordon [4]. In order to propagate in the anomalous dispersion regime with low loss, they used a mode-locked colour centre laser that was specifically constructed by Mollenauer to operate around 1.55 µm. The delivered 7-ps pulses were launched into a 700 m-long single-mode silica fiber, and the output was measured in the time domain through nonlinear autocorrelation, and in the frequency domain by an optical spectrometer (Fig. 2). At low power, the pulse showed temporal broadening (Fig. 2a), but at a critical peak power around 1 W, the initial temporal profile was maintained as the input pulse was a fundamental soliton (Fig. 2b). As the power was further increased, the temporal profile showed up to a factor of 3 temporal compression and spectral broadening as expected for a higher-order soliton (Fig. 2c). Ultimately this resulted in pulse splitting (Fig. 2d).
The pioneering experiments of Mollenauer, Stolen and Gordon involved distances of less than a kilometer, so the influence of losses was limited. However, if transcontinental distances were to be covered, it was essential to be able to compensate for losses at regular intervals, notably using optical amplifiers. Solitons proved to be a robust solution, and the 80s and 90s saw impressive technological demonstrations [5]. Nevertheless, after the burst of the internet bubble in the 2000s, alternative technologies such as coherent signal modulation proved to be more economically suitable for increasing data rates.

But beyond telecommunications, the soliton concept has proven central to advance the field of ultrafast optics. To cite just a few examples, soliton dynamics are central to octave spanning supercontinuum generation, and the generalised concept of the non-linear Schrödinger equation is a universal equation of physics that applies not only to fibre optics, but also to hydrodynamics, quantum or plasma physics. For single-mode fibres, this equation governs the longitudinal evolution of the complex envelope of the electrical field $\psi(z,t)$ for an optical pulse of a typical duration of several picoseconds:

$$i\frac{\partial \psi}{\partial z} - \beta_2 \frac{\partial^2 \psi}{\partial t^2} + \gamma |\psi|^2 \psi = 0$$

It includes the effect of second-order dispersion ($\beta_2$ coefficient), leading in the spectrum to a linear delay of the frequency components. This translates into a temporal redistribution of instantaneous frequencies (chirp), resulting in a temporal broadening. In the case of a fibre with anomalous dispersion ($\beta_2 < 0$), blue frequencies components are shifted towards the front of the pulse, while red ones are shifted towards the tail of the pulse. The Kerr non-linearity (associated with the nonlinear coefficient $\gamma$) manifests itself as a self-phase modulation term that generates new frequencies, thus broadening the pulse spectrum. The generated frequencies are proportional to the temporal intensity gradient. Therefore, for a bright pulse, red is generated on the leading edge and blue on the trailing edge. For a fundamental soliton (the right shape with the right input power), the changes induced by self-phase modulation can exactly cancel the effect of dispersion such that the pulse preserves its temporal and spectral characteristics. This condition results in the following simple relation for the critical peak power: $P = 3.11 |\beta_2| / (\gamma T_p^3)$, where $T_p$ is the pulse duration (at half-maximum).

![Figure](image.png)

**Figure**. Soliton basis: (a) Evolution of the temporal and spectral content of a Gaussian pulse when submitted to dispersive or nonlinear effects exclusively. (b) The perfect balance achieved in the case of the soliton where the instantaneous frequency changes induced by linear and nonlinear effects perfectly cancel each other.
dissipative soliton has enabled the development of new classes of modelocked fibre lasers with dramatically increased power and pulse quality. In addition, transposed to the field of integrated waveguides and micro-resonators, soliton concepts underpin our models of the generation of optical frequency combs and associated applications in metrology.

CONCLUSION
The soliton concept proposed in 1973 and confirmed by experiments in 1980 has opened up major fundamental and applied perspectives that are still relevant today. The many studies of soliton physics over the last 50 years provide a remarkable illustration of how fruitful it can be to combine knowledge from different fields of physics. In particular, by exploiting the extremely low level of loss when compared to other physical media, fibres have now become the test bed of choice for the study of a much wider range of nonlinear phenomena, including analogies with event horizon physics and the formation of oceanic rogue waves. The soliton also provides a stunning example of how an initial problem developed in an industrial context can inspire a new paradigm in the fundamental field of complex non-linear dynamics. Research related to solitons and their applications shows no sign of slowing down, and soliton concepts remain central to the most modern applications of ultrafast photonics.

REFERENCES