

BACK TO BASICS:

Computer-generated holograms

From the invention of the laser and early calculators to the very latest applications

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Diffractive optical elements (DOEs) have many advantages over conventional ones. For a start, they are generally lighter, take up less room, and have a wider variety of properties. They can also be used to fabricate hybrid elements that modify polarization, compensate for dispersion, and so on. Moreover, they are potentially low cost, as they can be mass replicated using manufacturing methods similar to those employed for semiconductor devices. It should therefore come as no surprise to learn that DOEs already have many industrial applications.

The rapid development of photolithographic techniques, and micromanufacturing in general, means that it is now possible to produce increasingly precise and complex DOEs, or phase masks. When these are placed in front of a light beam, they spatially modify the wave front to produce the desired light distribution.

There are hundreds of industrial applications for DOEs in areas ranging from optical telecommunications, lasers, anti-counterfeit solutions, microdisplay technologies and optical design, to industrial vision, lithography, biotechnology, and a whole host of gadgets.

For example, a red laser diode can be fitted with interchangeable DOE modules (red cylinder, *Fig. 1*) to create dot grids, concentric circles, lines, and so on. These types of tools are extremely useful for industrial vision and three-dimensional measures.

Spatial light modulators (SLMs), microelectromechanical systems (MEMs) and micromirror devices, or deformable mirrors, are also becoming increasingly diversified and efficient. All these DOEs provide local wavefront control and make it possible to

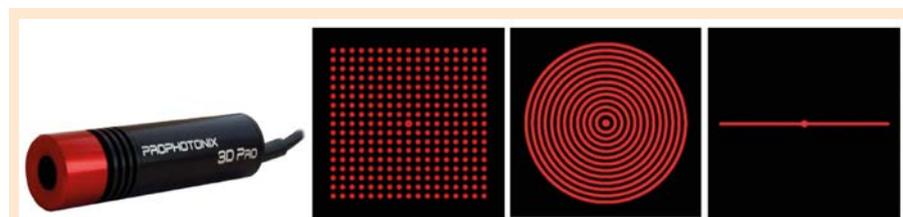


Figure 1. Examples of figures obtained with diffractive elements (www.prophotonix.com).

modify light beams in real time. They are being used in a growing number of sophisticated optical applications.

Before describing some of their recent and more spectacular research applications, we recall how these phase masks are calculated.

Digital holography, prehistory

First, we will travel back 35 years in time to the yellow corridor of Building 503 at Orsay University, where Institut d'Optique Graduate School (IOGS) students did their lab work. For generations of young SupOpticians, stretching back to the 1980s, one of the most memorable labwork sessions was undoubtedly the one on computer-generated holograms (CGHs). When I was appointed in 1992

to teach these practicals at the institute, I soon found that this experiment was an endless source of fascination for our engineering students. Back then, the holograms were calculated using a very imposing HP 1000 minicomputer! If I remember rightly, students would feed in a 512×512 binary table, in other words a black-and-white image measuring 512×512 pixels, which the faithful HP 1000 would then store in its memory, and calculate its 2D fast Fourier transform (FFT). Using Lohmann's coding scheme (to which we will return later), the 2D FFT allowed the computer to represent the binary table as an array of 512 complex numbers. The resulting Lohmann hologram was printed in the building's ground-floor computer room. Printed? Well, in reality, the *printer* was probably

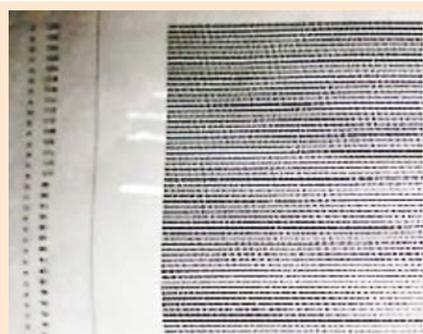


Figure 2. A Lohmann hologram, probably printed in around 1985 at the Optics Institute.

an HP plotter. “A machine that made as much noise as a pneumatic drill”, recalls Jean-Paul Hugonin, who ran the same type of practical with first-year PhD students. “We might as well have been carving out the holograms with a chisel. It was like something out of the Stone Age”, adds Thierry Avignon, who always went round before each session checking that none of the plotter’s black pens had dried out! Jean Taboury, who was director of studies back then and jointly in charge of the physical optics lectures with Pierre Chauvel, has sent me a photo of a CGH dating from this heroic period (Fig. 2).

Once they had printed out this sheet containing apparently randomly distributed tiny black squares, the students then had to photographically reduce this hologram. This involved taping the A4 sheet to the wall, shining old studio lights on it, carrying out highly precise manual focusing, and taking a dozen or so photographs of this sheet, adjusting the diaphragm aperture and shutter speed to obtain a 24×36 mm negative with as much contrast as possible. To achieve sufficient resolution, we used an excellent 50-mm fixed focal length lens mounted on a Canon camera body.

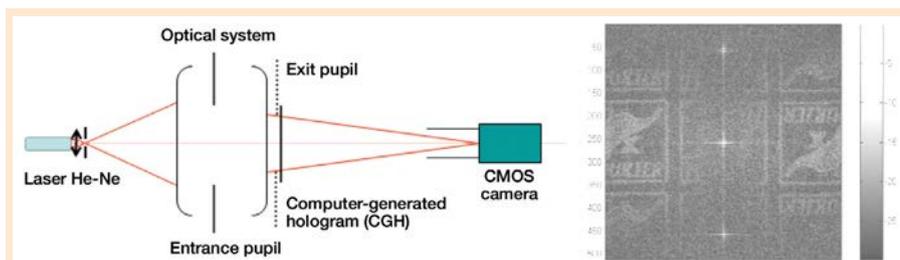


Figure 3. Reconstruction of a Lohmann hologram using an optical bench and bicorn image captured by the camera.

Then came the moment that was the most tense and nerve-racking for the students, when they had to develop the film. After taking out the 50-ASA black-and-white film (always very fine-grained, to ensure the best resolution) in the dark room prepared beforehand by Marie-Thé Plantegenest or Thierry Avignon, they would have to roll it onto a reel (no easy task) and dip it in a whole series of baths. Only then could they start feeling positive about the negatives! This operation was not always successful, and the poor students sometimes had to start all over again. For many of them, this was undoubtedly a lastingly traumatic experience. It must have been so for Riad Haidar, editor-in-chief of *Photoniques*, but also an alumnus of SupOptique who presumably has spent some time in this quaint dark room back in 1999, otherwise why would he have commissioned me to write this article so many years later?

But let us get back to our homemade CGH. Once the film had been developed, everyone could relax, and the students would take a break while the film dried in the old grey cupboard housing the hot-air dryer. Once the film was ready, the best method was to carefully cut it into individual photos and surround the reduced hologram to ensure that the light passed through – and not around – the hologram.

At last came the reward, when the CGH, or more accurately the photo-reduced negative of the hologram (Fig. 2), was placed in a convergent HeNe laser beam, and an image such as this one (Fig. 3) appeared in the convergence plane, that is, the image plane of the beam waist.



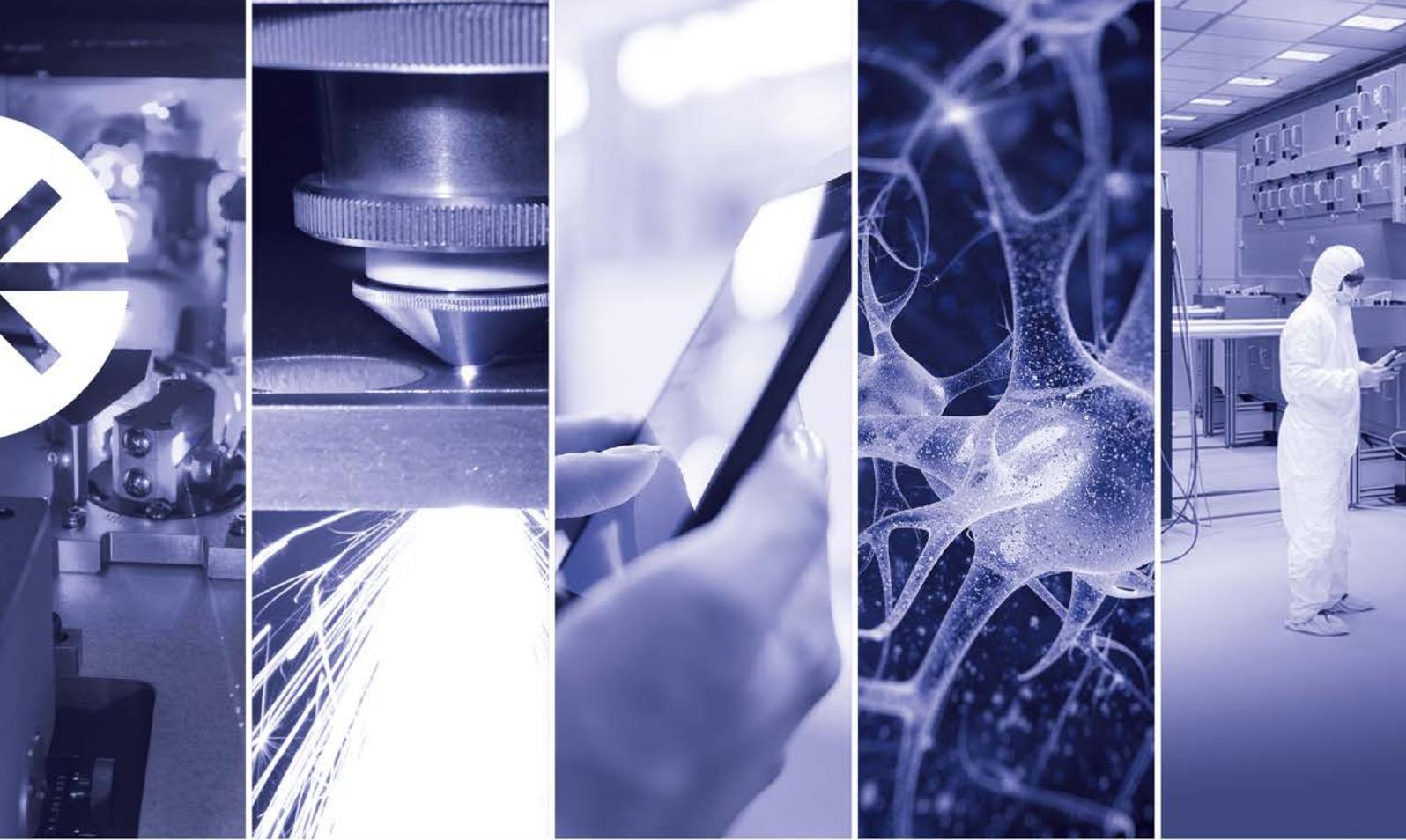
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Figure 4. The Cheshire cat (128 × 128 pixels) and a small part of its CGH.

Indeed, at the end of his annual lecture on Joseph Fourier to first-year students at École polytechnique, Philippe Grangier, a friend since our student days at ENS Cachan, would present this experiment to illustrate the FT. Each year, in the month of June, Thierry Avignon and Cédric Lejeune would take the legendary optical bench by car from the Optics Institute building to École polytechnique and set it up in the lofty Henri Poincaré lecture hall. And when Philippe revealed the result of the optical FT of a hologram of the Polytechnician’s bicorn hat (Fig. 3) on the panoramic screen, there would be a rapturous round of applause for the experiment, Philippe’s lecture, the genius of Joseph Fourier and, I assume, the famous bicorn.

Fourier’s optics

For it is indeed FT that we are talking about here, as the illuminance in the image plane of a point source (coherent lighting) is an example of Fraunhofer diffraction, the key being to observe at the point conjugate to the point source. The wave amplitude in the image plane (camera plane) can therefore easily be calculated from the wave amplitude in the pupil using 2D FFT. Conversely, to obtain an image of the object of your dreams in the camera, however, you simply need to calculate the FT of that image and modulate the wave in the pupil plane through transmittance proportional to this 2D FFT.

Simply? That is precisely the problem, as transmittance is a 2D complex matrix. You therefore need an SLM that can modulate amplitude, and an SLM that can modulate phase, in order to give the wave in the pupil the right phase and amplitude distribution, and thus produce the image of your dreams in the image plane.

You might think that a black-and-white negative cannot provide phase modulation, but the bicorn on the large screen says otherwise. CGHs therefore do modulate the wave phase. But how do we get round the absence of a phase modulator?

The secret of the Lohmann hologram

Denis Gabor’s brilliant invention of holography in 1947, which he hoped would improve the resolution of electron microscopes, may provide the key. To conserve the phase of an optical wave (object wave), this object wave must be made to interfere with a reference wave. Once the interference fringes pattern has been printed on the hologram, we can reconstruct the object wave simply by illuminating the hologram with the reference wave.

The first sentence in the acceptance speech given by Denis Gabor 24 years later, when he was awarded the Nobel Prize in 1971 for his invention of holography was as follows [1]:

“I have the advantage in this lecture, over many of my predecessors, that I need not write down a single equation or show an abstract graph. One can of course introduce almost any amount of mathematics into holography, but the essentials can be explained and understood from physical argument.”

Following his example, can we indeed explain without a single line of calculus the secret of Lohmann’s hologram, which he set out in an article

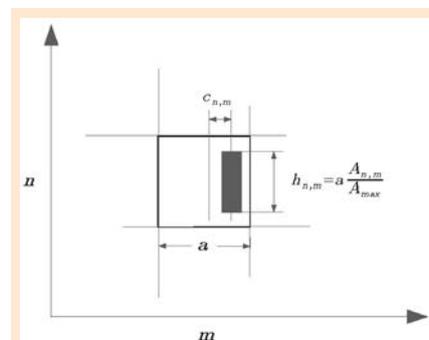


Figure 5. The phase detour illustrated on a square pixel with sides of length a , at the position (n, m) . The height $h_{n,m}$ of the rectangle is proportional to the amplitude of this pixel of the FFT $A_{n,m}$. The displacement of the rectangle $c_{n,m}$ is proportional to its phase.

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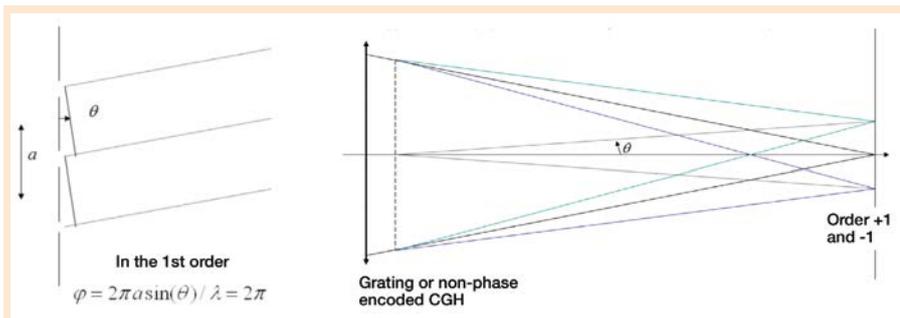


Figure 6. Case of a non-phase encoded CGH (equidistant white rectangles). The diffraction orders are obtained from a Foucault test card pattern transmittance.

published in 1967 [2], entitled “Binary Fraunhofer Holograms, Generated by Computer”. This secret is the so-called detour phase method that Lohmann had described a year earlier [3]. In his article he wrote: “There are three possibilities to influence the phase of a light wave: retardation while traveling through a dielectric, phase jump at reflection, and detour phase.”

We should say first of all that even though the CGH provides a flat image (and not a 3D one), it is still a hologram, as it provides a means of phase encoding.

Here is a typical hologram that can be produced with this method (Fig. 4). You will notice that I have chosen a cat – not necessarily Schrödinger’s cat, both dead and alive at the same time, even if its hologram is reminiscent of a quantum cryptography system. In actual fact, it is a Cheshire cat, which can appear and disappear at will – an ability that greatly amused Alice in her *Adventures in Wonderland*.

Each tiny white rectangle encodes one of the pixels in the cat’s 2D FFT (128 × 128 pixels) (Fig. 4), based on the following rule:

- the rectangle’s surface area is proportional to the amplitude of the corresponding complex number;
- the rectangle is horizontally displaced relative to the pixel’s centre, this displacement being proportional to the phase of the corresponding complex number.

Amplitude coding is easy enough to understand, as the amplitude of the wave transmitted by each rectangle is proportional to the surface of the

tiny transparent rectangle on the film. Phase coding is slightly more subtle. If the phase is zero for all the pixels, all the rectangles will be vertically aligned, and the CGH’s FT will only produce a point at the centre of the image (zero diffraction order) and horizontally aligned points (including ±1st diffraction orders). If we obtain a +1st order point in the direction θ , it is constructive interference, and the waves from two successive white squares will therefore be in phase (the phase is equal to 2π) (Fig. 6).

If, on the other hand, the rectangles are horizontally displaced relative to the centres of the pixels, the phase of the waves from two successive white squares will be proportional to the distance between these squares (Fig. 7), and we will obtain a picture of the cat centred on the direction of the

+1st order. In -1st order, we would, of course, obtain a symmetrical image of the cat, as we would have encoded a phase with the opposite sign! The FT of the conjugate of an object yields a symmetrical image, as we can clearly see for the bicorns in Figure 3.

In reality, there are other ways of making our cat appear. One method consists in performing the FT of an image that is symmetrical in relation to its centre (Fig. 8), in which case the 2D FFT is a matrix of real numbers, the phase is zero, and the white rectangles therefore do not need to be displaced.

Lastly, we have said nothing about a method that can be systematically used, whereby a random phase is added to the binary object before calculating its 2D FFT, in order to achieve uniform amplitude in the Fourier plane (Fig. 9). This random phase has no impact on the reconstructed image, as the human eye and the camera (along with any other optical detector) are sensitive to the squared modulus of the wave, not its phase. The idea for this particular method, like others, comes from what we normally think of as holograms, and it does not work for objects with low light scattering. The wave from the object has to be spread across a large area of the hologram, so we place a finely ground glass plate in its trajectory, just in front of the object.

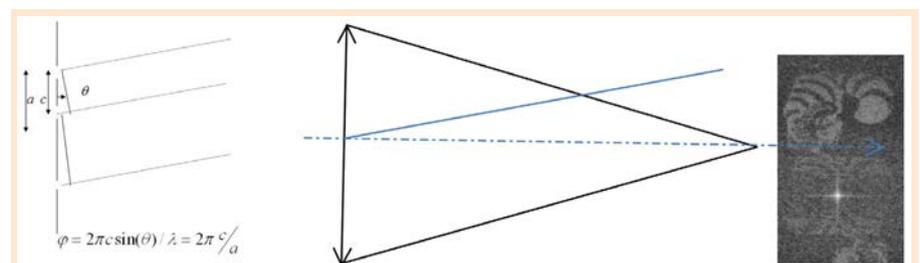


Figure 7. Case of a phase-coded CGH (white rectangles displaced proportionally to the phase) and result of the optical Fourier transform.

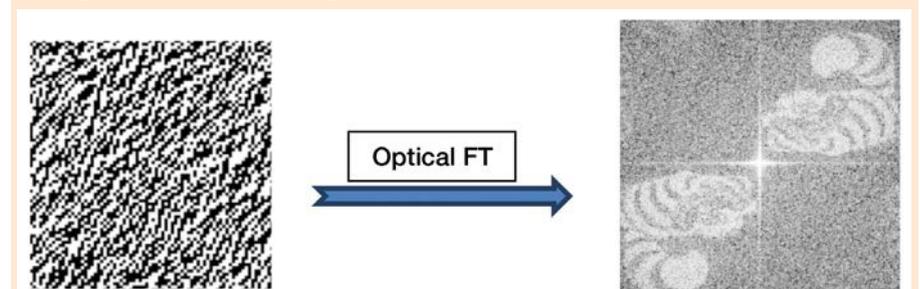


Figure 8. Binary hologram of symmetrical cats and reconstruction by the optical FT.

Digital holography today

Even though the Labwork Department has changed its name to Experimental Teaching Laboratory (LEnSE) and is now led by Fabienne Bernard, a colleague of mine who joined the IOGS in 1996, and even though the labwork session is no longer officially about CGHs, but about simple diffractive elements, it is still run for 3rd-year students, who continue to be amazed when they see cats appearing

on the walls. It now takes no time at all to produce a CGH. The holograms are calculated with a short MATLAB programme on a standard PC, and a laser printer then transfers them onto transparencies. We also study holographic aberration correction, as CGHs or diffractive devices are still used to measure aspherical surfaces or correct aberrations.

These CGHs, with a spatial amplitude modulation of the transmitted wave, are highly inefficient. The illumination is inevitably at its greatest

in the axis of the image plane (zero diffraction order) and orders +1 and -1 are far darker.

As I indicated at the start of this article, technological advances mean that extremely accurate phase masks, or DOEs, can now be mass-produced. The images in *Figure 1* show that these DOEs allow to obtaining almost no zero order. The good news is that amplitude modulation is not required. Phase modulation is all that is needed to achieve any binary light distributions. The phase can be retrieved

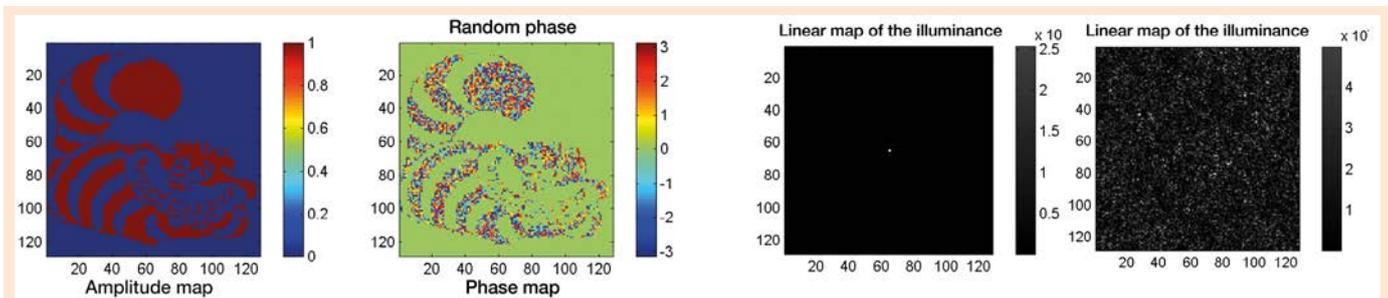


Figure 9. Solitary cat, and cat multiplied by random phase (left); FFT of the cat without and with random phase (right).

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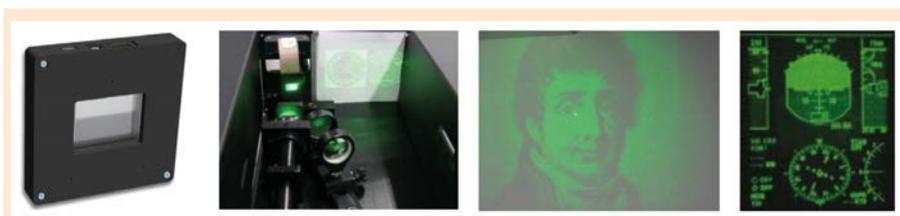


Figure 10. SLM Holoeye and portrait of Joseph Fourier by Julien Léopold Boilly (www.holoeye.com).

using the Gerchberg-Saxton iterative 2D FFT algorithm [5], which is extremely easy to programme in MATLAB. Moreover, some phase modulators can now be configured to operate at video rate (real time). These are liquid crystal display (LCD)-based SLMs, which have been used since the early 2000s in mobile phones and video projectors to modulate intensity. If a different configuration of the input and output polarizers is used, these same devices become phase modulators. Their advantage over DOEs is that they allow the CGH to be modified so that, for example, vehicle speed can be displayed on a dashboard and, just 50 ms later, the portrait of Joseph Fourier (Fig. 10).

LEnsE's recent acquisition¹ of an SLM Holoeye enabled master's student Ivan Fernandez de Jauregui Ruiz to demonstrate a superb experiment involving the selection of spatial modes injected into a weakly multimode fibre, as part of his Erasmus Mundus programme.

This particular type of SLM is also widely used in microscopy (e.g. most recently in structure illumination methods, bringing a twofold improvement over the diffraction limit) or in optical tweezers that can trap and handle cells or particles (*Photoniques* 66, July-August 2013, "La pince optique" by Jean-Pierre Galaup).

SLMs can also be used to trap atoms, as demonstrated by Sylvain Ravets, a member of the Quantum

Optics group of the Charles Fabry lab. When Sylvain reprised research carried out with the assistance of

LEnsE back in 2004 [5], as part of his PhD supervised by Antoine Browaeys [6] he successfully designed microtrap arrays for single atoms with varied and complex geometries (Fig. 11).

To end this article, which gave me such pleasure to write, I will simply quote the sentence that SupOptique students find on the LEsE computer screen each time they quit one of my MATLAB applications: "Fourier optics is beautiful. Thank you and goodbye!" ■

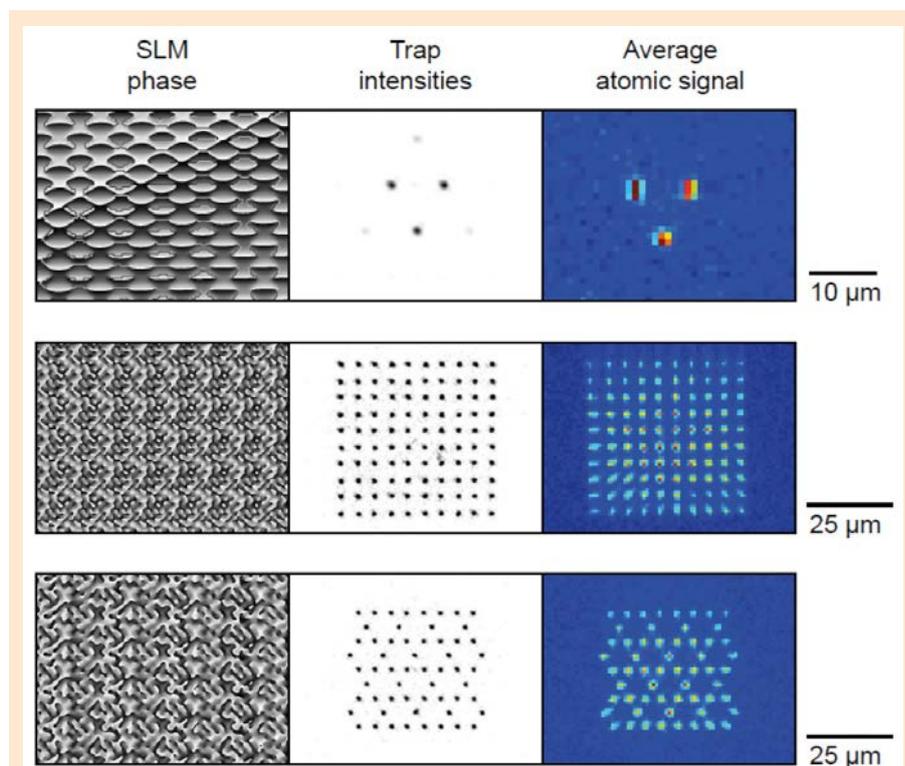


Figure 11. Examples of microtrap arrays (left: hologram; centre: CCD images of traps; right: mean of 1000 fluorescence images of single charged atoms in the traps) (thesis defended by Sylvain Ravets on 18 December 2014 at IOGS and [6]).

FURTHER READING

- [1] "Holography", Nobel Lecture, December 11, 1971 by Dennis Gabor, Imperial College of Science and Technology, London.
- [2] "Binary Fraunhofer Holograms, Generated by Computer", A.W. Lohmann and D.P. Paris, *Applied Optics* 6(10), 1739 (1967)
- [3] "Complex spatial filtering with binary masks", B.R. Brown and A.W. Lohmann, *Applied Optics* 5(6), 96 (1966)
- [4] "A practical algorithm for the determination of phase from image and diffraction plane pictures", R.W. Gerchberg and W.O. Saxton, *Optik* 35, 237 (1972)
- [5] "Holographic generation of microtrap arrays for single atoms by use of a programmable phase modulator", S. Bergamini, B. Darquié, M. Jones, L. Jacubowicz, A. Browaeys, and P. Grangier, *Journal of the Optical Society of America B* 21, 1889 (2004)
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